Impacts of Packet Scheduling and Packet Loss Distribution on FEC Performances: Observations and Recommendations

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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Abstract: Forward Error Correction (FEC) is commonly used for content broadcasting. The performance of the FEC codes largely vary, depending in particular on the object size and on the number of parity packets produced, and these parameters have already been studied in detail by the community. However the FEC performances are also largely dependent on the packet scheduling used during transmission and on the loss pattern introduced by the channel. Therefore this work analyzes their impacts on three FEC codes: LDGM Staircase, LDGM Triangle, two large block FEC codes, and Reed-Solomon. Thanks to this analysis, we define several recommendations on how to best use these codes, depending on the test case and on the channel, which turns out to be of utmost importance.

Key-words: Multicast, Forward Error Correction (FEC), LDPC, Reed-Solomon, Loss Pattern, Packet Scheduling

Impacts de l'Ordonnancement des Paquets et de la Distribution des Pertes de Paquets sur les Performances de trois Codes Correcteurs d'Erreurs: Observations et Recommandations

Résumé : Les codes correcteurs d'erreurs (ou FEC, "Forward Error Correction") sont largement utillisés dans le contexte de la diffusion à large échelle de contenus. Les performances de ces codes FEC dépendent fortement de la taille de l'objet encodé et du nombre de paquets de parité produits. Ces deux paramètres ont déjà été étudiés en détails par la communauté. Toutefois les performances dépendent aussi fortement de l'ordonnancement des paquets utilisé lors de la transmission ainsi que de la distribution des pertes sur le canal de transmission. Par conséquent, ce travail analyse les impacts de ces paramètres sur trois codes : LDGM Staircase et LDGM Triangle, deux codes FEC de type grands blocs, et Reed-Solomon. Grace à cette analyse, nous établissons des recommandations d'une très grande importance pratique pour une utilisation optimale de ces codes en fonction du canal de transmission et du scénario d'utilisation.

Mots-clés : Multicast, Codes Correcteurs d'Erreurs, LDPC, Reed-Solomon, Distribution des Pertes de Paquets, Ordonnancement des Paquets

1 Introduction

1.1 Context of the work

This work analyzes the impacts of packet scheduling in the context of a content delivery systems like ”IP Datacast” (IPDC) [12, 6] in DVB-H, the ”Multimedia Broadcast/Multicast Service” (MBMS) [1] in 3GPP, or data broadcast to cars (e.g. [5]). These systems are characterized by the fact that there is no back channel, and therefore no repeat request mechanism can be used that would enable the source to adapt its transmission according to the feedback information sent by the receiver(s). The lack of feedback channel however enables an unlimited scalability in terms of number of receivers, who behave in a completely asynchronous way. Using a reliable multicast transmission protocol like ALC [9], along with the FLUTE [13] file delivery application, can turn out to be highly effective in this context [5].

Yet, in order to be efficient, these approaches largely rely on the use of a Forward Error Correction (FEC) scheme running at the application layer, within the ALC reliable transport protocol (we motivate the use of FEC in section 4.2). The channel is therefore a “packet erasure channel” and packets either arrive (with no error) or are lost (e.g. because of router congestion problems). After an FEC encoding of the content, redundant data is transmitted along with the original data. Thanks to this redundancy, up to a certain number of missing packets can be recovered at the receiver. The great advantage of using FEC with multicast or broadcast transmissions is that the same parity packet can recover different lost packets at different receivers.

Since we only consider file delivery applications in this work, the transmission latency has little importance, which would not be true with streaming applications. Therefore we will not consider the potential impacts of FEC codes and transmission scheme on the decoding latency at a receiver.

1.2 Goals of the work

The performance of the FEC code is largely impacted by the transmission scheduling. For instance, sending all source packets first and then parity packets does not necessarily yield the same efficiency as sending the packets in random order. The packet loss distribution observed by a receiver also largely impact the decoding performances and a given transmission scheme may yield good results for a specific loss distribution and yield catastrophic results in other circumstances. This work analyzes the impacts of packet scheduling and loss behaviors on the performances of three FEC codes: Reed-Solomon, LDGM Staircase and LDGM Triangle. Thanks to this analysis, we define several recommendations on how to best use these codes, which turns out to be of utmost practical importance. For instance it enables to optimize the FLUTE session to a specific download environment, or on the opposite, to find transmission schemes that will behave correctly (but may be not optimally) in a wide set of different environments.

In this work we do not considered FEC codes who are known to be covered by IPRs and for which no public domain, open-source, implementation exists. In particular we will not consider Tornado™ and Raptor™ codes [4]. This deliberate choice is likely to enlarge the usefulness of our results to a large part of the community who prefer free codes.

The remainder of the paper is organized as follows: we first introduce the three FEC codes; section 3 explains and motivates the modeling method we used; section 4 presents and analyzes the performance of several transmission schemes while section 5 does the same with a reception model; finally section 6 explains how to use these results in practice, then we conclude.

2 Introduction to RSE, LDGM Staircase and LDGM Triangle Codes

2.1 Terminology

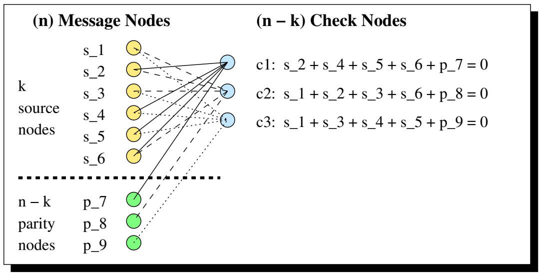
FEC encoding of an object produces redundant data. Thanks to this redundancy, up to a certain number of missing packets can be recovered at the receiver. More precisely k *source packets* (A.K.A. data packets) are encoded into n packets (A.K.A. encoding packets). The additional $n - k$ packets are called *parity packets* (A.K.A. FEC or redundancy packets). A receiver can then recover the k source packets provided it receives any k packets (or a little bit more than k with LDGM/LDPC codes) out of the n possible. The *FEC expansion ratio* is the $\frac{n}{k}$ ratio and it defines the amount of parity packets produced. It is the inverse of the *code rate* (i.e. $\frac{k}{n}$). In the present paper we will only consider the FEC expansion ratio terminology.

2.2 RSE Code

The Reed-Solomon erasure code (RSE) is one of the most popular FEC codes. RSE is intrinsically limited by the Galois Field it uses [14]. A typical example is GF(2^8) where $n \leq 256$. With one kilobyte packets, a FEC codec producing as many parity packets as data packets (i.e. $n = 2k$) operates on blocks of size 128 kilobytes at most, and all files exceeding this threshold must be segmented into several blocks, which reduces the global packet erasure recovery efficiency (e.g. if B blocks are required, a given parity packet has a probability $1/B$ to recover a given erasure, and $B = 1$ is then the optimal solution). This phenomenon is known as the “Coupon Collector Problem” [3]. Another drawback is a huge encoding/decoding time with large (k, n) values, which is the reason why GF(2^8) is preferred to GF(2^{16}) in spite of its limitations on the block size. Yet RSE is optimal (it is an MDS code) because a receiver can recover erasures as soon as it has received *exactly* k packets out of n for a given block.

2.3 LDGM Codes

We now consider another class of FEC codes that completely departs from RSE: Low Density Generator Matrix (LDGM) codes, that are variants of the well known LDPC codes introduced by Gallager in the 1960s [7].



(a) Bipartite graph

$$[\,H_1 \mid Id_3\,] = \begin{array}{c} \begin{array}{cccccc|ccc} s_1 & & \dots & & & s_6 & p_7 & \dots & p_9 \end{array} \\ \left(\begin{array}{cccccc|ccc} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \end{array} \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array}$$

(b) H matrix

Figure 1: A regular bipartite graph and its associated parity check matrix for LDGM.

2.3.1 Principles

LDGM codes rely on a bipartite graph between left nodes, called *message nodes*, and right nodes, called *check nodes* (A.K.A. constraint nodes). The k source packets form the first k message nodes, while the parity packets form the remaining $n - k$ message nodes. The upper part of this graph is built following an appropriate left and right degree distribution (in our work the left degree is 3). The lower part of this graph follows other rules that depend on the variant of LDGM considered (e.g. with LDGM, figure 1 (a), there is a bijection between parity and check nodes). This graph creates a system of $n - k$ linear equations (one per check node) of n variables (source and parity packets).

A dual representation consists in building a parity check matrix, H. With LDGM, this matrix is the concatenation of matrix H_1 and an identity matrix I_{n-k}. There is a 1 in the $\{i; j\}$ entry of matrix H each time there is an edge between message node j and check node i in the associated bipartite graph.

Thanks to this structure, encoding is extremely fast: each parity packet is equal to the sum of all source packets in the associated equation. For instance, packet p_7 is equal to the sum: $s_2 \oplus s_4 \oplus s_5 \oplus s_6$. Besides LDPC/LDGM codes can operate on *very large blocks*: several hundreds of MBytes are common. However LDGM is not an MDS code and it introduces a decoding inefficiency: $inef\_ratio * k$ packets, with $inef\_ratio \geq 1$, must be received for decoding to be successful. The $inef\_ratio$, experimentally evaluated, is therefore a key performance metric.

2.3.2 Iterative Decoding Algorithm

With LDGM, there is no way to know in advance how many packets must be received before decoding is successful (LDGM is not an MDS code). Decoding is performed step by step, after each packet arrival, and may be stopped at any time.

The algorithm is simple: we have a set of $n-k$ linear equations of n variables (source and parity packets). As such this system cannot be solved and we need to receive packets from the network. Each non duplicated incoming packet contains the value of the associated variable, so we replace this variable in all linear equations in which it appears. If one of the equations has only one remaining unknown variable, then its value is that of the constant term. We then replace this variable by its value in all remaining equations and reiterate, recursively. As we approach the end of decoding, incoming packets tend to trigger the decoding of several packets, until all of the k source packets have been recovered.

2.3.3 LDGM Staircase Code

This trivial variant, suggested in [10], only differs from LDGM by the fact that the I_{n-k} matrix is replaced by a “staircase matrix” of the same size. This small variation affects neither encoding, which remains a simple and highly efficient process, nor decoding, which follows the same algorithm. But this simple variation *largely improves the FEC code efficiency*.

2.3.4 LDGM Triangle Code

In this variant of LDGM Staircase, the triangle beneath the staircase diagonal is now filled, following an appropriate rule [15]. This rule adds a “progressive” dependency between check nodes, as shown in figure 2. This variation further increases performance in some situations, while keeping encoding highly efficient (even if a bit slower since there are more "1"s per row). Here also decoding follows the same iterative algorithm.

Interested readers are invited to refer to [15]. An open source, GNU/LGPL implementation of these codes is also available at [11].

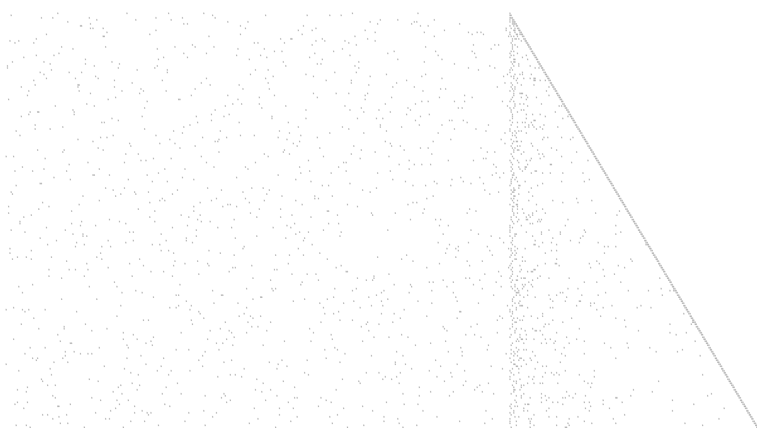


Figure 2: Parity check matrix (H) for LDGM Triangle (k=400, n=600).

3 Modeling of the Whole System

3.1 The Three Models

Let's imagine that a server wants to broadcast a big file using a large scale content delivery system using FLUTE. The content is first FEC encoded which produces additional parity packets (we motivate the use of FEC in section 4.2). The sender must now decide in which order source and parity packets will be sent, and this choice will largely impact the whole system performances as we will see later. This packet scheduling constitutes the transmission model.

The channel is characterized by a packet loss distribution. It may be a lossy channel with long bursts of packet erasures, or it may be a channel where losses are completely independent from one another. This packet loss distribution constitutes the loss model.

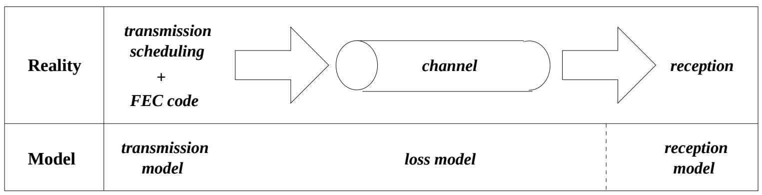


Figure 3: Modeling methods.

These two transmission and loss models together characterize a reception behavior at a receiver (Figure 3). In case of broadcast applications, there is no reason that different receivers experience the same loss model, so the reception behavior is receiver dependent. This is the approach that will be used for most experimental evaluations (section 4).

The reception behavior may also be modeled the other way round, by providing a reception model. This model defines which packets are received (and when) by a receiver. This approach can be complementary to the use of the transmission and loss models, for instance to study the FEC code performances in controlled situations. This is the approach that will be used in section 5.

3.2 The Channel Model

Finding an appropriate error model for a given channel is a complex task, especially with wireless networks where it is difficult to take into account all parameters (e.g. channel fading, reflections, refractions, diffractions, Doppler effects). Yet in this work we are only interested in a packet loss model (rather than a bit error model) that provides a high level abstraction of the channel parameters. The well known two state Markov model (A.K.A. Gilbert model) is such a simplified loss model, and it is widely used in the literature [2, 16].

The model is composed of two states: the *no-loss* state where no packet loss occurs, and the *loss* state where packets are lost (figure 4). p indicates the probability to go from no-loss state to loss state, and q from loss state to no-loss state. Having p and q we can calculate the global packet error

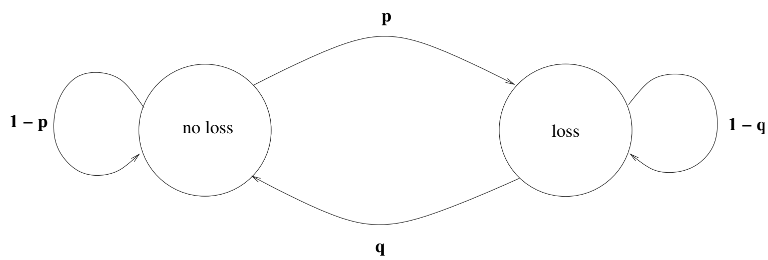


Figure 4: Two state Markov loss model.

probability [2]:

$$p_{global} = \frac{p}{p+q}$$

This probability is represented as a 3D-graph in figure 5.

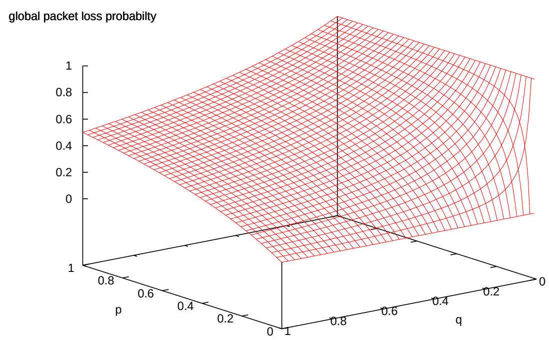


Figure 5: Global loss probability.

Depending on the channel modeled, the p and q values will differ. For a given channel it may be possible to determine p and q using packet loss traces. For instance, this has been done in [8] for traces coming from an GSM channel, and in [16] for traces coming from end-to-end connections in the Internet.

The Gilbert model also covers some specific loss behaviors that we want to emphasize:

- No loss: This perfect channel corresponds to $p = 0$.
- Independent and Identically Distributed (IID) losses (A.K.A. the Bernoulli model): This memoryless channel corresponds to $q = 1 - p$.

We are aware that the Gilbert model has some shortcomings in error modeling accuracy [8, 16]. We believe that it is however sufficient for our work since it already covers a very large set of loss behaviors. Moreover we took care to perform experiments with a very large set of p and q values (14×14 grid) in order to cover as many channel behaviors as possible. Other more complex models (e.g. the n-state Markov models), that may be required for specific channels, will be considered in future works.

When is Decoding Impossible?

In our work we want to know, given a certain FEC code, how many losses a receiver can support. With the Gilbert model we can calculate the maximum number of packet losses supported by any FEC code. Let's consider a FEC code producing $n-k$ parity packets from k source packets. We then transmit $n_{sent} \leq n$ packets over the network (sending all packets is not mandatory, see section 5). The number of packets actually received is equal to:

$$n_{received} = n_{sent} * (1 - p_{global}) \tag{1}$$

$n_{received}$ must be at least equal to $inef\_ratio * k$ (remember that LDGM codes are not MDS) for decoding to be successful. When $n_{received} = inef\_ratio * k$ we have:

$$q = \frac{-p * inef\_ratio}{inef\_ratio - \frac{n_{sent}}{k}}$$

These limits are shown in figure 6 for FEC expansion ratios 1.5 and 2.5, and assuming $inef\_ratio = 1$ (which is a lower bound for $inef\_ratio$). This figure shows that several areas of the (p, q) parameter space cannot enable a receiver to decode the object. This is not a flaw of the FEC code, it's merely a fundamental limitation.

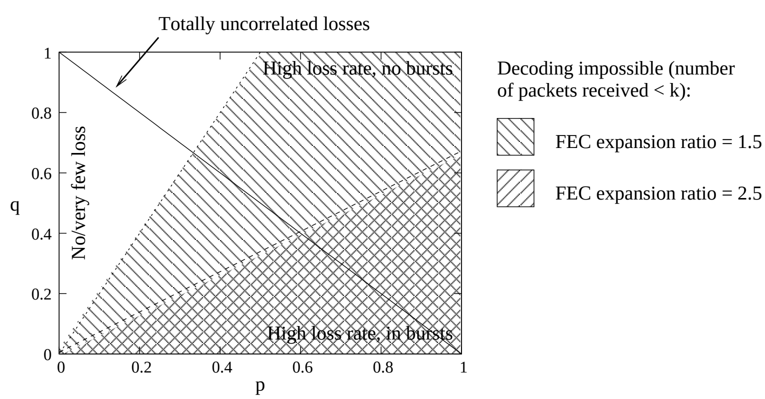


Figure 6: Loss limits.

4 Simulation Results With Six Transmission Models

4.1 Methodology

The methodology used to conduct performance tests is the following. We considered the RSE, LDGM Staircase and LDGM Triangle codes, and two typical FEC expansion ratios: 1.5 and 2.5 (they correspond to code rates $2/3$ and $2/5$ respectively). The object is composed of 20000 packets (i.e. $k = 20000$ with LDGM-* codes).

The performance metric used is the average inefficiency ratio: $inef\_ratio = \frac{n_{necessary\_for\_decoding}}{k}$, which is the total number of packets received when decoding completes, divided by the number of source packets. Of course the optimal value is 1.0, and the higher this $inef\_ratio$, the more packets are needed in excess to k for decoding, which is not good[1].

We considered six different *transmission models*. Although this is not an exhaustive study, we believe that these transmission schemes already cover a large set of possibilities and give good insights on the overall performances.

For each transmission model and FEC code, we study the impacts of the channel by varying the p and q probabilities in $[0; 1]$ (14×14 values are considered for $(p; q)$). Each point in the resulting 3-D graphs is the average $inef\_ratio$ value over 100 simulations. Yet if decoding fails for any of the 100 tests, we do not plot any point. This strict strategy enables us to better highlight the areas where the decoding probability is not acceptable. The numerical results of our simulation for the most interesting transmission schemes and codes are reported in the Appendix of this paper.

For comparison purposes, we sometimes plot an additional curve, $\frac{n_{received}}{k}$, which corresponds to the total number of packets received (even after decoding stopped) divided by the number of source packets. This is the maximum value that the inefficiency ratio can achieve. It also gives an idea on the number of packets that a receiver may still receive after decoding ($n_{received} - n_{necessary\_for\_decoding}$). When $\frac{n_{received}}{k} = 1$ the curve corresponds to the limits described in section 3.2 and figure 6. Any additional packet loss will necessarily lead decoding to fail.

4.2 Why is FEC Needed?

To motivate the use of FEC, we did a small test. Rather than using FEC to recover losses, a sender may decide to transmit each packet x times. In figure 7 $x = 2$ and packets are sent in a random order. It shows that decoding is only possible with $p = 0$, and the average inefficiency ratio is then near 2.0 which means that the receiver waits almost systematically the end of the transmission to reconstruct the object. For all $p > 0$ at least one experiment failed, and therefore no inefficiency ratio is shown, as explained in section 4.1. This test highlights the *poor performances when using repetition only instead of FEC* for content broadcasting.

[1] With large block LDGM codes, an inefficiency ratio greater than 1.0 is caused by the non MDS nature of these codes, whereas with RSE, which is an MDS code, this is caused by the coupon collector problem (section 2.2). More information can be found in [15].

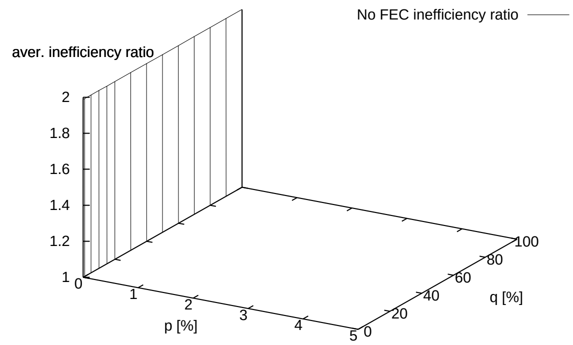


Figure 7: Performances without FEC but 2 repetitions.

4.3 Tx_model_1: Send Source Packets Sequentially, Then Parity Packets

This scheme consists in sending all source packets sequentially, and then all parity packets, also sequentially. The results are shown in figure 8 (the LDGM Staircase curves are not shown since results are similar to LDGM Triangle).

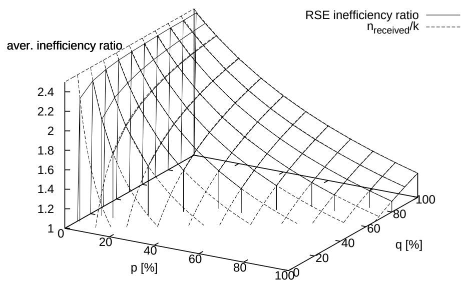
We notice a first obvious result: without loss ($p = 0$) the inefficiency ratio is 1.0 with all codes. Indeed, all source packets are received and the receiver does not need any of the following parity packets. We'll see that other transmission schemes do not necessarily have this good property.

With losses (in burst or not) all codes show a similar behavior. The inefficiency ratio curve is very close to the $\frac{n_{received}}{k}$ curve for nearly all values of p and q. It means that the receiver always needs to wait almost the end of transmission to reconstruct the object. With RSE this is not surprising since the object is segmented in blocks: if a source packet of the last block is lost, the receiver needs to wait the end of the transmission, when the associated parity packets will be sent.

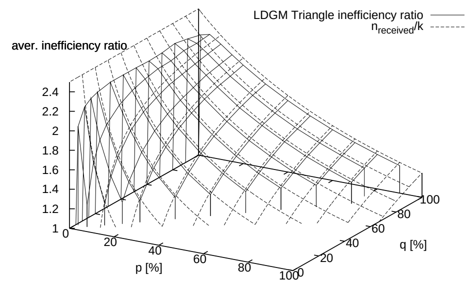
With LDGM codes this behavior is more surprising. These large block FEC codes encode the whole object directly, and parity packets are created using source packets coming from different parts of the content. However each parity packet depends on the previous one (because of the staircase in the parity check matrix, also present in LDGM Triangle codes). This dependency negatively impacts decoding performance when several sequential parity packets get lost.

Finally, RSE covers a smaller area (when $z - axis > 0$) than LDGM-* codes, which indicates that its erasure recovery capabilities are more limited. This is especially true for long packet loss bursts (small q). This is easy to understand: since packets are sent sequentially, and a single long burst results in the loss of a large number of packets of the same block. Decoding this block becomes difficult, especially with a small FEC expansion ratio (e.g. 1.5).

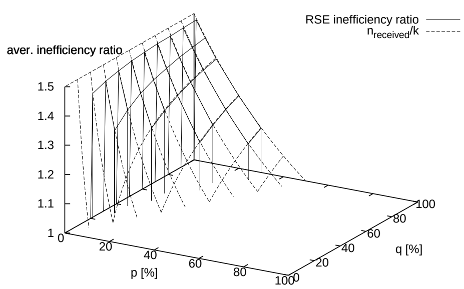
For all these reasons this transmission model is definitively bad, which was relatively foreseeable.



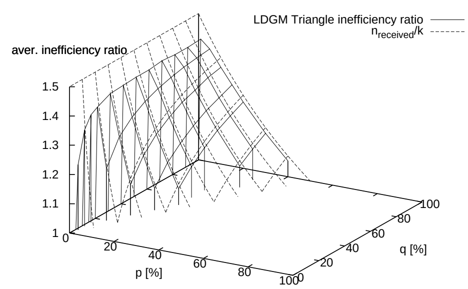
(a) RSE and a FEC expansion ratio of 2.5.



(b) LDGM Triangle and a FEC expansion ratio of 2.5.



(c) RSE and a FEC expansion ratio of 1.5.



(d) LDGM Triangle and a FEC expansion ratio of 1.5.

Figure 8: Tx_model_1

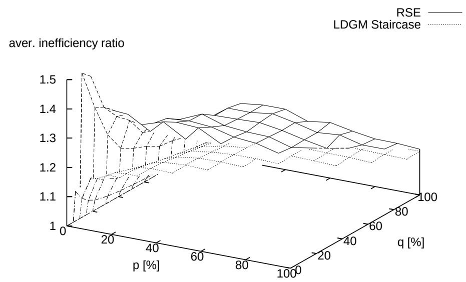
4.4 Tx_model_2: Send Source Packets Sequentially, Then Parity Packets Randomly

One idea to counteract the bad results of Tx_model_1 is to transmit the parity packets in a random order rather than sequentially. For RSE the advantage is clear: parity packets of the last few blocks can be transmitted earlier, so the receiver has on average less to wait before receiving parity packets for any block. This is confirmed in figures 9(a) and 9(c). The inefficiency ratio is much better than with Tx_model_1 and moreover this ratio is relatively constant.

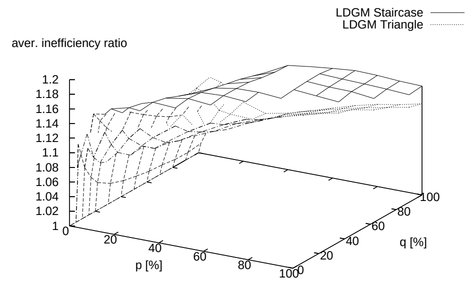
LDGM Triangle codes exhibit relatively good performances and largely outperform RSE codes (figures 9(b) and 9(d)). Yet the LDGM Staircase 3D-curve has a hole around $p = 50$ and $q =$

70 (Figure 9(b)), where exactly one test failed. This is not acceptable in practice, and therefore Tx_model_2 is not recommended with LDGM Staircase and higher loss ratios. Conversely LDGM Staircase may be used with small loss ratios where the code yields very good results.

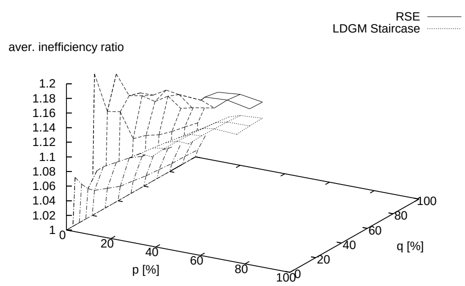
This transmission model confirms that with LDGM-* codes, parity packets should not be sent sequentially but randomly, to prevent the loss of sequences of parity packets. All the following tests will confirm this observation.



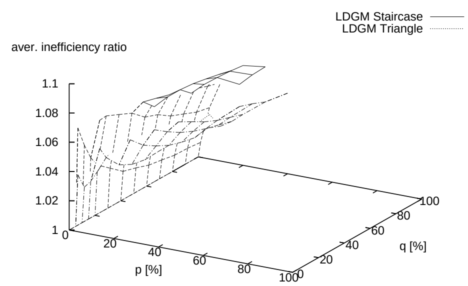
(a) RSE versus LDGM Staircase and a FEC expansion ratio of 2.5.



(b) LDGM Triangle vs. Staircase and a FEC expansion ratio of 2.5.



(c) RSE versus LDGM Staircase and a FEC expansion ratio of 1.5.



(d) LDGM Triangle vs. Staircase and a FEC expansion ratio of 1.5.

Figure 9: Tx_model_2.

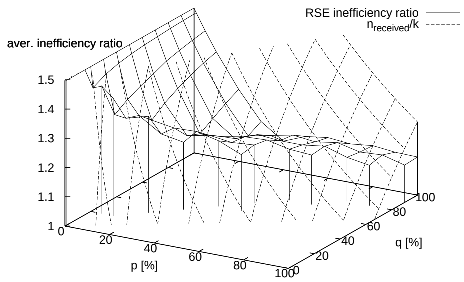
4.5 Tx_model_3: Send Parity Packets Sequentially, Then Source Packets Randomly

We now consider the case dual to Tx_model_2: we first transmit all parity packets sequentially, then we transmit the source packets either sequentially or randomly. We only consider the latter case in this paper since tests (not included in this paper) have shown that sending the source packets sequentially yields uninteresting results (it makes no difference with LDGM-* codes, and we experience a worse behavior with RSE).

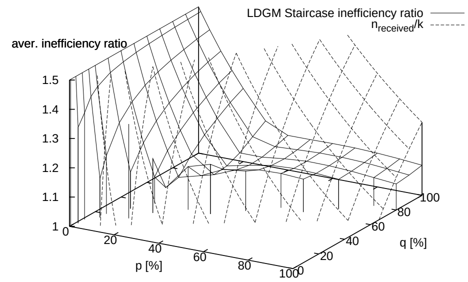
Non-systematic FEC codes can decode using only parity packets (no source packet). RSE can be used as a non-systematic code if the number parity packets is high enough (i.e. if $n - k \geq k$). But this is not the case with LDGM Triangle and Staircase that need at least a small number of data packets to start decoding.

Figure 10 shows that with $p = 0$, the LDGM-* codes need exactly one source packet to decode the content, and therefore the inefficiency ratio is ≈ 1.5 for a FEC expansion ratio of 2.5. With RSE, the receiver needs 29903 packets when $p = 0$, that is to say decoding is possible when k packets of the last block have been received. Therefore the inefficiency ratio is also ≈ 1.5 for a FEC expansion ratio of 2.5.

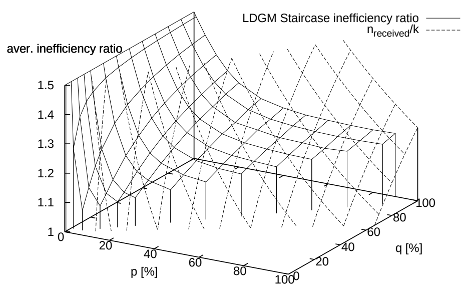
Globally, these tests show that performances are not that interesting, and this transmission scheme may only be interesting for some specific loss patterns.



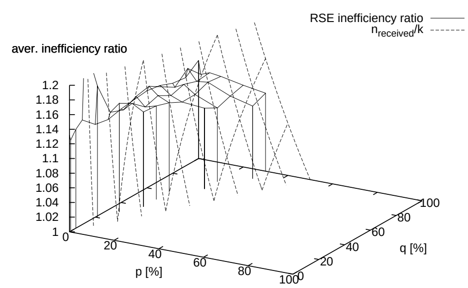
(a) RSE and a FEC expansion ratio of 2.5.



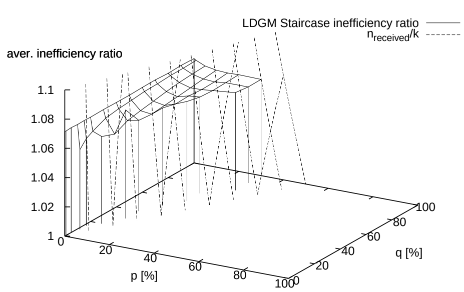
(b) LDGM Staircase and a FEC expansion ratio of 2.5.



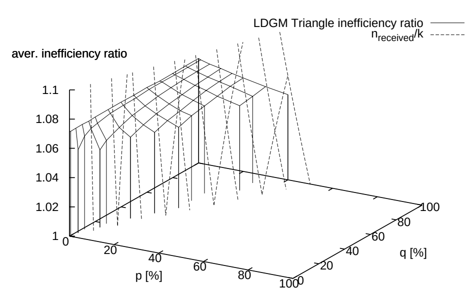
(c) LDGM Triangle and a FEC expansion ratio of 2.5.



(d) RSE and a FEC expansion ratio of 1.5.



(e) LDGM Staircase and a FEC expansion ratio of 1.5.



(f) LDGM Triangle and a FEC expansion ratio of 1.5.

Figure 10: Tx_model_3.

4.6 Tx_model_4: Send Everything Randomly

In this transmission model, source and parity packets are sent in a fully random order.

Results are shown in figure 11. RSE offers the worst performances with an inefficiency ratio around 1.25. LDGM Staircase performs better and offers a ratio of 1.15. Finally LDGM Triangle yields the best results with ratios going from 1.12 to 1.14.

For the RSE and LDGM Staircase codes, the performances are relatively stable, independently of the packet loss behavior. This is not true with LDGM Triangle codes, that show better results with smaller p_{global}. This observation can generally be made when LDGM Triangle sends it parity packets randomly: it achieves better inefficiency ratios with smaller p_{global}. On the opposite, LDGM Staircase codes are not sensitive to p_{global} in this case.

4.7 Tx_model_5: Interleaving (RSE only)

Packet interleaving is a commonly used solution with small block FEC codes like RSE to increase their robustness against packet erasure bursts. The idea is to spread the transmission of one block over an interval that is longer that the loss burst duration. The maximum distance between two packet transmissions of the same block is achieved by sending successively one packet of each block, until reaching the last block, and then continuing with the following packet of each block, and so on.

With LDGM codes this interleaving scheme is not feasible since there is only one block. What we call interleaving for LDGM consists in sending successively one source packet and $\frac{n}{k}$ parity packet (because the FEC expansion ratio is not necessarily equal to 2), and so one.

Results are shown in figure 12. As suspected, results are excellent with RSE, actually the best ones compared to all other transmission schemes. The packets are optimally aligned using interleaving, and for all loss patterns this solution give optimal results. This is not a surprise since interleaving has been intensively used along with RSE codes.

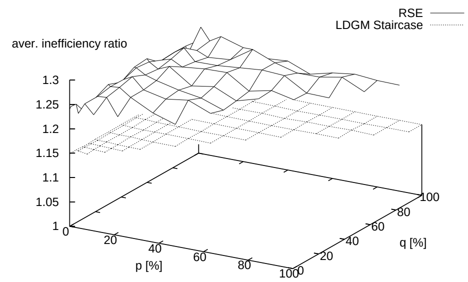
4.8 Tx_model_6: Send Randomly a Few Source Packets Plus Parity Packets

In this transmission model we transmit only a few source packets in addition to all parity packets. More precisely, we first pick randomly 20% source packets and schedule them randomly with all parity packets. This transmission model requires that the FEC expansion ratio be high enough (otherwise less than k packets will be received), and we chose 2.5.

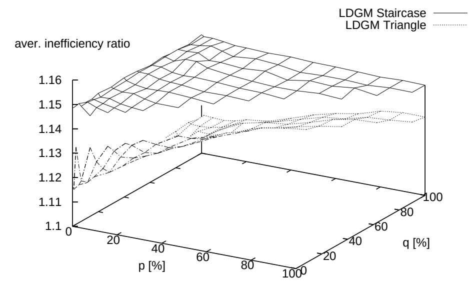
The results are shown in figure 13. If all codes have constant performances, LDGM Staircase largely outperform other codes. Note that the fact that LDGM Staircase performs better than Triangle is rather unusual.

5 Simulation Results With a Reception Model

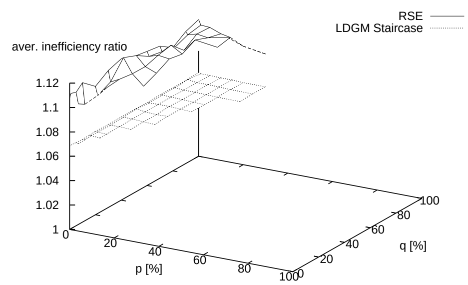
In this section we directly specify a reception model, without any consideration for the transmission and loss models that may generate it. The goal is to better analyze an FEC code performances in a completely controlled environment.



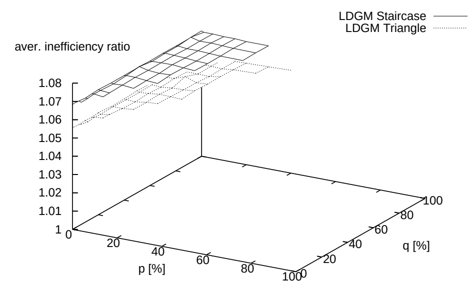
(a) RSE versus LDGM Staircase and a FEC expansion ratio of 2.5.



(b) LDGM Triangle vs. Staircase and a FEC expansion ratio of 2.5.



(c) RSE versus LDGM Staircase and a FEC expansion ratio of 1.5.



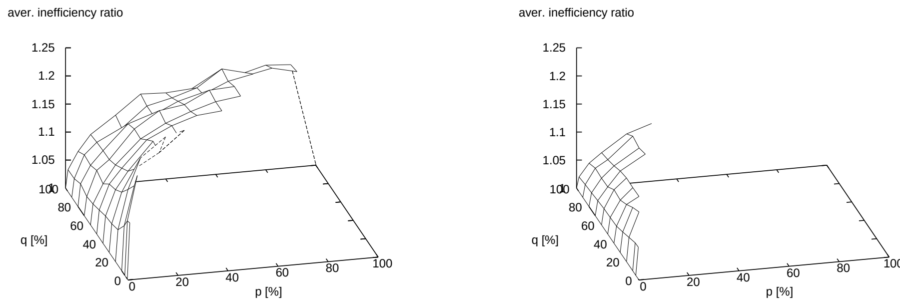
(d) LDGM Triangle vs. Staircase and a FEC expansion ratio of 1.5.

Figure 11: Tx_model_4

5.1 Rx_model_1: Receive a Few Source Packets, Then Parity Packets Randomly

Section 4.8 has shown that sending only a few source packets along with the parity packets may be interesting. In this reception model, we further study this phenomenon. The difference with Tx_model_6 is that we now guarantee that these source packets arrive and are used for decoding. To do so the receiver first gets the source packets, and then, randomly, all parity packets.

We only consider LDGM Staircase and FEC expansion ratio of 2.5, since section 4.8 has shown that it yields the best results. We analyze the performance of the FEC codes as a function of the



(a) RSE and a FEC expansion ratio of 2.5 (NB: the figure is rotated for better visibility).

(b) RSE and a FEC expansion ratio of 1.5 (NB: the figure is rotated for better visibility).

Figure 12: Tx_model_5

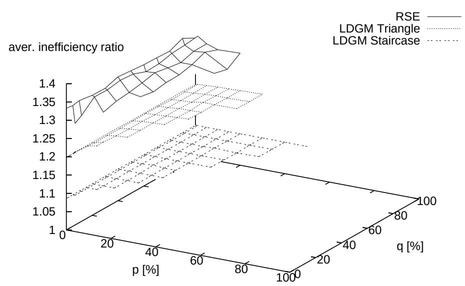


Figure 13: Tx_model_6 with LDGM Triangle, LDGM Staircase and RSE and a FEC expansion ratio of 2.5.

number of source packets received. Figure 14 shows that excellent performances are achieved when around 400 to 1000 source packets received. This is a very small number of packets compared to the object size ($k = 20000$ packets). Receiving more (or fewer) packets will degrade performances! To the best of our knowledge, we never saw such results mentioned in the literature, and we think this promising (and surprising) result deserves some complementary studies.

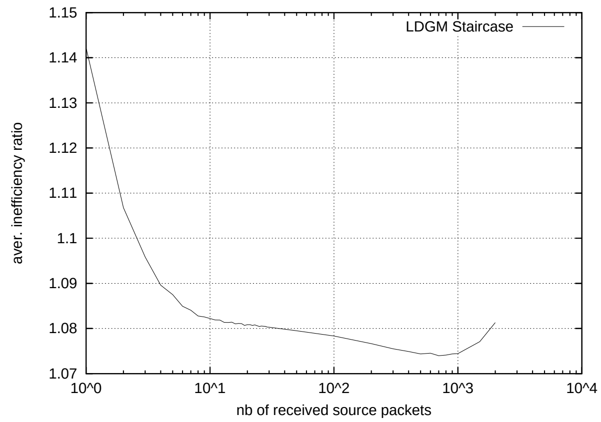


Figure 14: Rx_model_1 with LDGM Staircase.

6 Discussions and Recommendations

6.1 Summary of the Results

We now discuss the previous results and draw some recommendations. Regarding FEC code performances:

- RSE: RSE should always be used along with Interleaving (which is not a new result). Yet, even in that case, RSE shows lower performances than the best LDGM codes, for instance LDGM Triangle with Tx_model_2 or Tx_model_4.

- LDGM Triangle and LDGM Staircase: In most cases *LDGM Triangle yields better results*. There are some exceptions though, like with Tx_model_6, or Tx_model_2 with a low p value, where LDGM Staircase is more efficient. For both codes it makes no difference whether source packets are sent randomly or sequentially. However *sending parity packets sequentially must be avoided*, since loss bursts will severely degrade performances.

Regarding the transmission models:

- Tx_model_1 and Tx_model_3 are of little interest in all cases.

- *Packet interleaving (Tx_model_5) is unavoidable with RSE, no matter the loss model*.

- The following models, that use totally or partially random transmissions, give good results with LDGM-* codes: Tx_model_2, Tx_model_4, and Tx_model_6. More precisely:

  – Tx_model_2 is the preferred scheme for LDGM Triangle and LDGM Staircase *when there are few packet losses*. With LDGM Staircase great care must be taken of possible decoding failures with higher loss ratios.

– Tx_model_4 (along with LDGM Triangle) and Tx_model_6 (along with LDGM Staircase) are the schemes that are the less dependent on the loss distribution. Therefore they are the *preferred solutions when the loss model is unknown, and Tx_model_4 is preferred if, additionally, we suspect very high loss rates*.

6.2 In Practice

The previous results allow us to select an appropriate transmission scheme for a given use case. If the channel characteristics are unknown, LDGM Triangle and Tx_model_4 are excellent choices. If on the opposite the channel features (and its loss model) are known, we can compare all the FEC performances for all transmission schemes around the $(p; q)$ point, and make a choice.

The transmission can further be optimized by adapting n_{sent}, the number of packets actually sent. Remember that $n_{received} - n_{necessary\_for\_decoding}$ is the number of packets the receiver receives after decoding has finished. Our goal is now to have:

$$n_{received} = n_{necessary\_for\_decoding} + \epsilon \tag{2}$$

where ϵ is a small integer ($n_{received}$ must be a little bit greater than $n_{necessary\_for\_decoding}$ since some tolerance is required). By doing so, the number of packets received will be very close to the number of packets that are actually needed by a given receiver to successfully decode the object. This is achieved by reducing n_{sent} (Equation 1), that is to say by stopping transmissions after n_{sent} packets, without changing the scheduling. So the last $n - n_{sent}$ packets will never been sent.

Note that selecting a smaller FEC expansion ratio to reduce the number of packets sent in front of a certain loss model, is not always feasible, especially when there are only a small number of predefined ratios possible (e.g. because the codec has been optimized for some FEC expansion ratios). For instance, if only 1.5 and 2.5 ratios are possible, with RSE and Tx_model_5, a FEC expansion ratio of 2.5 is needed when $p = 40\%$. With 1.5, decoding would fail.

Besides we omit an essential performance metric: *encoding and decoding speed*. From this point of view, *LDGM codes are an order of magnitude faster than RSE codes*, as shown in our previous work [15]. This can be an essential criteria of choice when broadcasting big objects, for which the encoding and decoding times may be non negligible.

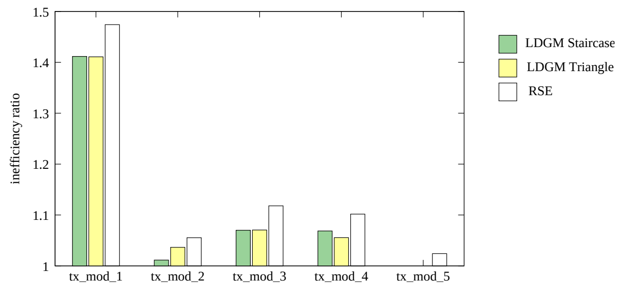
We now detail how to proceed in two specific use cases.

6.2.1 Homogeneous Receiver(s) and Known Channel

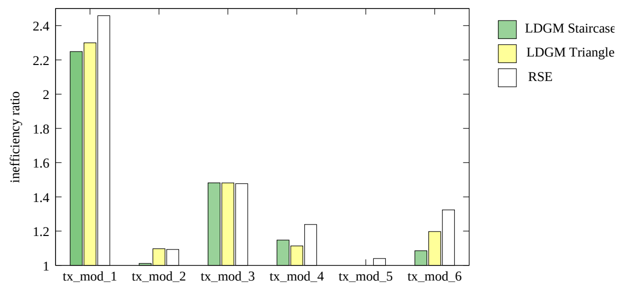
If there is only one receiver, or a set of receivers behind the same channel, the loss model may be identified and the p and q values determined (e.g. this can be the case when a content is broadcast in a cooperate network, or in some specific environments). Thanks to the p and q values, we can identify the best FEC code and transmission model for *this* environment.

Once they have been chosen, we can adapt n_{sent}. The inefficiency ratio is given by the associated curve. Then, according to formulas 1 and 2, the optimal n_{sent} is achieved when:

$$n_{sent} = \frac{n_{necessary\_for\_decoding}}{1 - p_{global}} \tag{3}$$



(a) FEC expansion ratio = 1.5



(b) FEC expansion ratio = 2.5

Figure 15: Example.

Let's consider the following example: a 50 MByte object is sent to a single receiver from Amhers Massachusetts to Los Angeles. [16] traced the loss behavior on this link and obtained $p = 0.0109$ and $q = 0.7915$ ($p_{global} = 0.0135$). Figure 15 shows the performances of the various FEC Code and transmission models, for the FEC expansion ratios 1.5 and 2.5. We clearly see that *Tx_model_2 with LDGM Staircase and a FEC expansion ratio of* 1.5 give the best results ($inef\_ratio \approx 1.011$).

We now calculate the optimal n_{sent} (formula 3):

$$n_{sent} = \frac{1.011 * 50MBytes}{1 - 0.0135} \approx 51.24MBytes$$

With 1024 byte packet payloads, it means that the sender may stop transmitting after ≈ 50041 packets. In order to add some tolerance to this result we can decide to set n_{sent} to 55000 packets. This is significantly less than the $n = 73243$ packets that would have been sent otherwise, while preserving transmission reliability.

6.2.2 Heterogeneous Receivers and/or Unknown Channel

Imagine now that an object is broadcast over a wireless channel. All receivers observe different packet loss distributions, depending on many external parameters (e.g. movement, obstacles, distance to the source).

If the loss pattern of each receiver is known, we may proceed as in the previous section, by choosing the (FEC code; transmission scheme; FEC expansion ratio) tuple that yields the best results for all receivers.

If the loss pattern of one or several receivers is unknown, which will probably be the general case, we need a universal scheme that shows the best possible performances in all situations. Sending everything randomly (Tx_model_4) and sending only a few parity packets (Tx_model_6) are the schemes that are the less dependent of the loss pattern, and are therefore the preferred solutions in this context for LDGM codes. All receivers then experience almost the same performances. RSE and interleaving is also possible, but performances (1) will largely differ between receivers, depending on their loss rate, and (2) will be lower for those that experience medium to high loss rates than with LDGM codes.

Here also, n_{sent} can be adapted when the loss patterns are known. Yet only a compromise is feasible: for each (p, q), we evaluate the inefficiency ratio and find the corresponding n_{sent} value (as explained in section 6.2.1); then we select the largest n_{sent} value.

7 Conclusions

This work investigated the impacts of packet scheduling and packet loss distributions on FEC performances, for content broadcasting applications. This work should be of great help for FLUTE-based file broadcasting systems, when there is no backward channel and where transmission reliability is achieved through the massive use of FEC and complementary techniques (e.g. cyclic transmissions within a carousel). Since we only consider file delivery systems, the transmission latency has little importance and large block FEC codes, like LDPC/LDGM codes, are good candidates.

The experiments carried out provide good insights on which (FEC code; FEC expansion ratio; packet transmission scheduling) tuples yield the best results for a given channel. Non surprisingly, experiments have shown that interleaving should always be used with RSE, no matter the loss pattern (which is not a new result). *Yet LDGM Staircase and Triangle codes usually perform significantly better than RSE*. For environments where the channel (and its packet loss distribution) is *unknown*, which is probably the general case, LDGM codes require a random transmission scheme: usually either (LDGM Triangle; Tx_model_4) or (LDGM Staircase; Tx_model_6). For environments where the loss distribution is *known*, our work helps to identify the best (FEC code; transmission scheme; FEC expansion ratio) tuple. Our results are therefore of utmost practical importance for optimizing the broadcasting system.

Of course these results omit an essential performance metric: encoding and decoding speed. From this point of view, *LDGM codes are an order of magnitude faster* than RSE codes, as shown in our previous work [15]. This can be an essential criteria of choice when broadcasting very large objects, for which the encoding and decoding times is non negligible. Said differently, LDGM codes are more suited to devices with small processing power than RSE which relies on complex mathematical operations.

Future works will naturally consist in studying new transmission schemes and new FEC codes. More elaborated channel models may also prove to be useful for specific target environments. Other performance metrics will also be added, like the maximum memory requirements needed in each case.

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Appendix: Numerical Experimental Results

p \ q	*0*	*1*	*5*	*10*	*15*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	-	-	1.081	1.103	1.103	1.112	1.097	1.104	1.095	1.094	1.095	1.097	1.090	1.078
5	-	-	1.124	1.087	1.074	1.070	1.082	1.095	1.100	1.104	1.092	1.083	1.102	1.106
10	-	-	-	1.124	1.102	1.086	1.072	1.075	1.079	1.080	1.088	1.089	1.093	1.102
15	-	-	-	-	1.124	1.108	1.088	1.075	1.072	1.071	1.075	1.062	1.077	1.089
20	-	-	-	-	-	1.125	1.102	1.086	1.078	1.074	1.069	1.071	1.074	1.081
30	-	-	-	-	-	-	1.124	1.106	1.096	1.087	1.079	1.076	1.073	1.071
40	-	-	-	-	-	-	-	1.124	1.112	1.103	1.094	1.087	1.082	1.077
50	-	-	-	-	-	-	-	-	1.125	1.114	1.106	1.101	1.094	1.086
60	-	-	-	-	-	-	-	-	-	1.124	1.116	1.109	1.103	1.096
70	-	-	-	-	-	-	-	-	-	1.132	1.124	1.116	1.111	1.105
80	-	-	-	-	-	-	-	-	-	-	1.131	1.125	1.118	1.112
90	-	-	-	-	-	-	-	-	-	-	-	1.131	1.124	1.118
100	-	-	-	-	-	-	-	-	-	-	-	-	1.130	1.125

Table 1: Tx_model_2: LDGM Triangle, FEC expansion ratio = 2.5

p \ q	*0*	*1*	*5*	*10*	*15*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	-	-	1.107	1.070	1.052	1.040	1.029	1.022	1.019	1.015	1.014	1.011	1.011	1.013
5	-	-	-	1.146	1.132	1.117	1.095	1.080	1.068	1.060	1.053	1.048	1.043	1.040
10	-	-	-	1.148	1.151	1.146	1.131	1.118	1.106	1.095	1.087	1.078	1.074	1.070
15	-	-	-	-	1.148	1.150	1.146	1.137	1.127	1.118	1.110	1.101	1.097	1.090
20	-	-	-	-	-	1.149	1.151	1.146	1.139	1.133	1.125	1.118	1.112	1.106
30	-	-	-	-	-	-	1.149	1.151	1.150	1.146	1.142	1.138	1.132	1.127
40	-	-	-	-	-	-	-	1.148	1.151	1.151	1.150	1.146	1.143	1.143
50	-	-	-	-	-	-	-	-	1.149	1.152	-	-	-	1.147
60	-	-	-	-	-	-	-	-	-	1.149	1.151	1.152	1.153	1.150
70	-	-	-	-	-	-	-	-	-	-	1.148	1.150	1.151	1.153
80	-	-	-	-	-	-	-	-	-	-	1.146	1.150	1.150	1.152
90	-	-	-	-	-	-	-	-	-	-	-	1.146	1.149	1.150
100	-	-	-	-	-	-	-	-	-	-	-	-	1.147	1.149

Table 2: Tx_model_2: LDGM Staircase, FEC expansion ratio = 2.5

p \ q	*0*	*1*	*5*	*10*	*15*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	-	-	1.035	1.025	1.026	1.030	1.038	1.035	1.039	1.039	1.035	1.036	1.035	1.035
5	-	-	-	-	1.050	1.041	1.031	1.026	1.024	1.025	1.027	1.027	1.029	1.030
10	-	-	-	-	-	-	1.050	1.041	1.035	1.031	1.028	1.026	1.028	1.024
15	-	-	-	-	-	-	-	1.053	1.047	1.041	1.037	1.034	1.031	1.029
20	-	-	-	-	-	-	-	-	1.055	1.050	1.045	1.041	1.038	1.035
30	-	-	-	-	-	-	-	-	-	-	-	1.053	1.050	1.046
40	-	-	-	-	-	-	-	-	-	-	-	-	-	1.055
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-
70	-	-	-	-	-	-	-	-	-	-	-	-	-	-
80	-	-	-	-	-	-	-	-	-	-	-	-	-	-
90	-	-	-	-	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 3: Tx_model_2: LDGM Triangle 1.5

p \ q	*0*	*1*	*5*	*10*	*15*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	-	-	1.068	1.053	1.042	1.035	1.028	1.020	1.018	1.015	1.013	1.011	1.011	1.010
5	-	-	-	-	1.069	1.069	1.065	1.061	1.054	1.050	1.044	1.041	1.037	1.035
10	-	-	-	-	-	-	-	1.070	1.068	1.065	1.062	1.059	1.056	1.054
15	-	-	-	-	-	-	-	1.069	1.070	1.070	1.069	1.068	1.066	1.063
20	-	-	-	-	-	-	-	-	-	1.069	1.070	1.070	1.069	1.068
30	-	-	-	-	-	-	-	-	-	-	-	1.068	1.070	1.070
40	-	-	-	-	-	-	-	-	-	-	-	-	-	-
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-
70	-	-	-	-	-	-	-	-	-	-	-	-	-	-
80	-	-	-	-	-	-	-	-	-	-	-	-	-	-
90	-	-	-	-	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 4: Tx_model_2: LDGM Staircase, FEC expansion ratio = 1.5

p \ q	*0*	*1*	*5*	*10*	*15*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.116	1.115	1.116	1.115	1.115	1.115	1.115	1.116	1.115	1.115	1.115	1.115	1.116	1.114
1	-	1.132	1.117	1.115	1.116	1.115	1.115	1.115	1.115	1.115	1.115	1.113	1.115	1.116
5	-	-	1.132	1.124	1.120	1.117	1.116	1.116	1.116	1.116	1.115	1.112	1.115	1.115
10	-	-	-	1.132	1.128	1.124	1.121	1.119	1.117	1.116	1.116	1.117	1.115	1.115
15	-	-	-	-	1.132	1.130	1.124	1.121	1.119	1.118	1.117	1.116	1.116	1.116
20	-	-	-	-	-	1.133	1.128	1.124	1.121	1.119	1.120	1.119	1.118	1.117
30	-	-	-	-	-	-	1.133	1.129	1.126	1.124	1.122	1.123	1.120	1.118
40	-	-	-	-	-	-	-	1.132	1.130	1.127	1.126	1.125	1.123	1.121
50	-	-	-	-	-	-	-	-	1.133	1.131	1.128	1.127	1.126	1.124
60	-	-	-	-	-	-	-	-	-	1.133	1.130	1.129	1.128	1.127
70	-	-	-	-	-	-	-	-	-	1.134	1.132	1.132	1.129	1.128
80	-	-	-	-	-	-	-	-	-	-	1.134	1.134	1.132	1.131
90	-	-	-	-	-	-	-	-	-	-	-	1.134	1.132	1.132
100	-	-	-	-	-	-	-	-	-	-	-	-	1.133	1.132

Table 5: Tx_model_4: LDGM Triangle, FEC expansion ratio = 2.5

p \ q	*0*	*1*	*5*	*10*	*15*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.056	1.056	1.055	1.056	1.055	1.056	1.055	1.055	1.056	1.055	1.056	1.055	1.056	1.056
1	-	-	1.056	1.055	1.056	1.055	1.055	1.055	1.055	1.055	1.056	1.055	1.055	1.056
5	-	-	-	-	1.056	1.056	1.055	1.055	1.055	1.055	1.056	1.055	1.056	1.056
10	-	-	-	-	-	-	1.056	1.056	1.056	1.056	1.058	1.055	1.056	1.055
15	-	-	-	-	-	-	-	1.056	1.056	1.056	1.056	1.055	1.055	1.055
20	-	-	-	-	-	-	-	-	1.056	1.056	1.056	1.056	1.056	1.056
30	-	-	-	-	-	-	-	-	-	-	-	-	1.056	1.056
40	-	-	-	-	-	-	-	-	-	-	-	-	-	1.056
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-
70	-	-	-	-	-	-	-	-	-	-	-	-	-	-
80	-	-	-	-	-	-	-	-	-	-	-	-	-	-
90	-	-	-	-	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 6: Tx_model_4: LDGM Triangle, FEC expansion ratio = 1.5

p \ q	*0*	*1*	*5*	*10*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	-	1.100	1.097	1.080	1.056	1.051	1.048	1.042	1.037	1.034	1.040	1.033	1.032
5	-	-	1.176	1.149	1.127	1.105	1.093	1.087	1.071	1.079	1.071	1.074	1.063
10	-	-	-	-	1.181	1.144	1.124	1.113	1.103	1.096	1.095	1.094	1.092
20	-	-	-	-	1.214	1.170	1.174	1.160	1.145	1.147	1.139	1.115	1.122
30	-	-	-	-	-	1.205	1.179	1.181	1.169	1.175	1.151	1.151	1.155
40	-	-	-	-	-	-	-	1.195	1.186	1.182	1.171	1.161	1.154
50	-	-	-	-	-	-	-	1.199	1.199	1.203	1.179	1.175	1.156
60	-	-	-	-	-	-	-	-	1.205	1.206	1.199	1.204	1.174
70	-	-	-	-	-	-	-	-	-	-	1.208	1.188	1.175
80	-	-	-	-	-	-	-	-	-	-	-	1.198	1.187
90	-	-	-	-	-	-	-	-	-	-	-	1.187	1.183
100	-	-	-	-	-	-	-	-	-	-	-	-	1.002

Table 7: Tx_model_5: RSE, FEC expansion ratio = 2.5

p \ q	*0*	*1*	*5*	*10*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	-	-	1.050	1.049	1.043	1.036	1.030	1.029	1.028	1.026	1.024	1.022	1.020
5	-	-	-	-	1.087	1.078	1.067	1.058	1.061	1.049	1.048	1.050	1.042
10	-	-	-	-	-	-	1.079	1.079	1.079	1.075	1.068	1.063	1.059
20	-	-	-	-	-	-	-	-	-	1.102	1.096	1.101	1.089
30	-	-	-	-	-	-	-	-	-	-	-	-	1.103
40	-	-	-	-	-	-	-	-	-	-	-	-	-
50	-	-	-	-	-	-	-	-	-	-	-	-	-
60	-	-	-	-	-	-	-	-	-	-	-	-	-
70	-	-	-	-	-	-	-	-	-	-	-	-	-
80	-	-	-	-	-	-	-	-	-	-	-	-	-
90	-	-	-	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 8: Tx_model_5: RSE, FEC expansion ratio = 1.5

In these tables, a $-$ sign means that at least one of the 100 test failed, i.e. the initial object could not be decoded from the packets received.

p \ q	*0*	*1*	*5*	*10*	*15*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*
0	1.086	1.086	1.086	1.086	1.086	1.086	1.086	1.086	1.085	1.086	1.086	1.086	1.086	1.086
1	-	-	1.086	1.086	1.086	1.086	1.086	1.086	1.086	1.086	1.086	1.085	1.086	1.087
5	-	-	-	-	1.086	1.086	1.086	1.087	1.086	1.086	1.086	1.085	1.086	1.086
10	-	-	-	-	-	1.086	1.087	1.086	1.089	1.086	1.086	1.086	1.086	1.086
15	-	-	-	-	-	-	1.086	1.086	1.086	1.086	1.086	1.085	1.086	1.086
20	-	-	-	-	-	-	-	1.086	1.086	1.086	1.086	1.087	1.086	1.086
30	-	-	-	-	-	-	-	-	-	1.086	1.086	1.085	1.086	1.086
40	-	-	-	-	-	-	-	-	-	-	-	1.087	1.087	1.086
50	-	-	-	-	-	-	-	-	-	-	-	-	-	1.086
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-
70	-	-	-	-	-	-	-	-	-	-	-	-	-	-
80	-	-	-	-	-	-	-	-	-	-	-	-	-	-
90	-	-	-	-	-	-	-	-	-	-	-	-	-	-
100	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 9: Tx_model_6: LDGM Staircase, FEC expansion ratio = 2.5

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